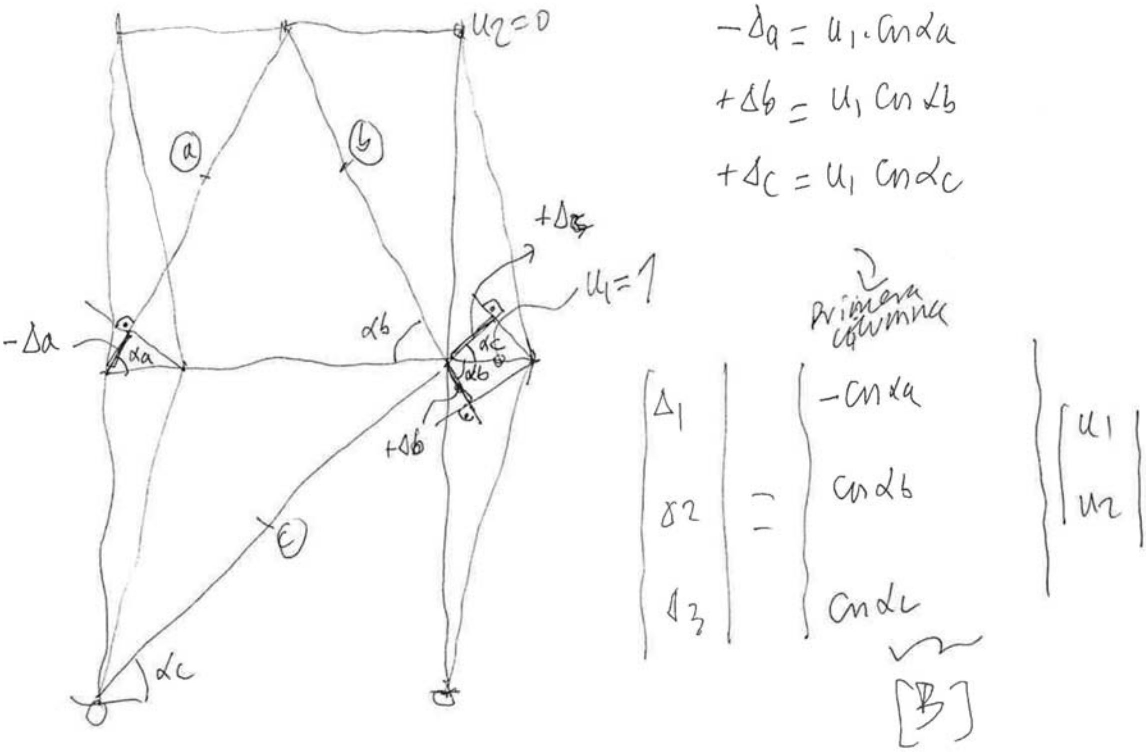
ETS de ARQUITECTURA de MADRID,
UNIVERSIDAD POLITÉCNICA DE MADRID

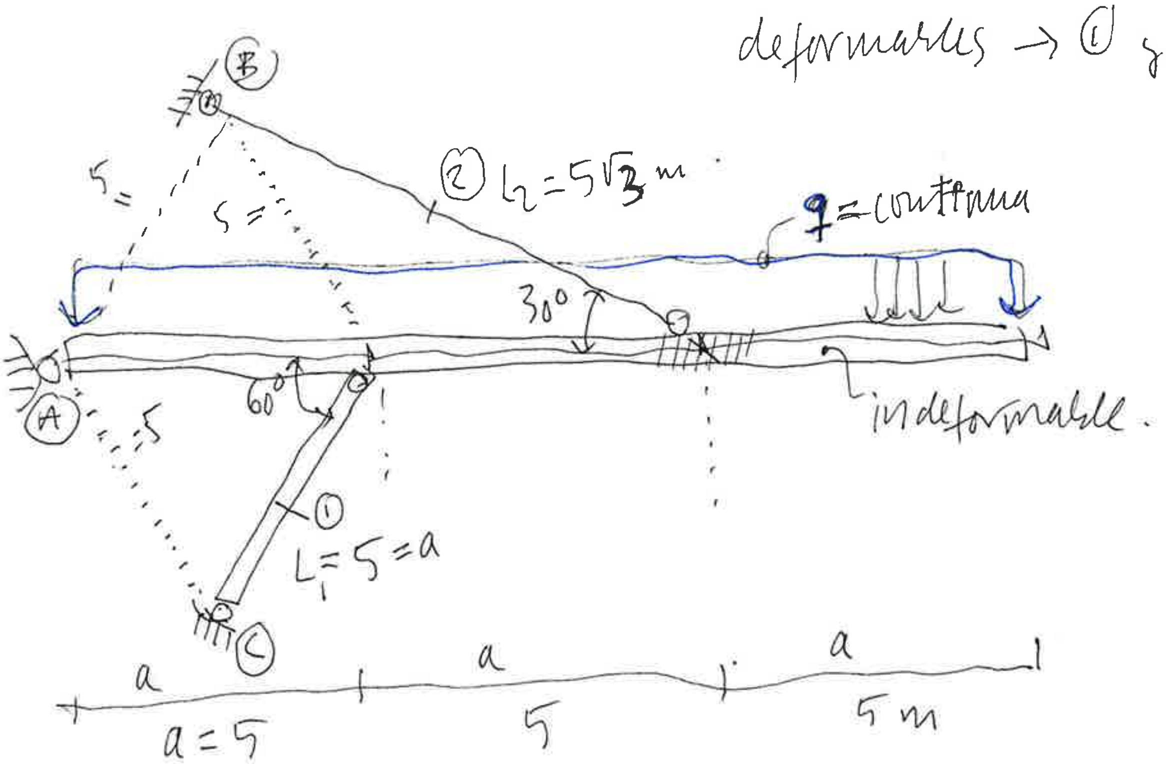
ESTRUCTURAS I: *EJERCICIOS DE APLICACIÓN DEL* ***MÉTODO BÁSICO DE LA RIGIDEZ***



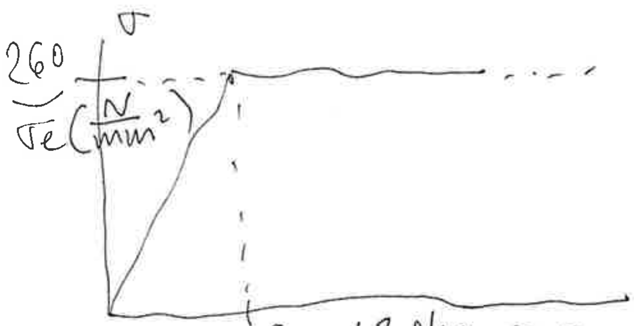
JOSÉ L. FERNÁNDEZ CABO

MADRID, Marzo 2012 (v1)

deformables → ① y ②

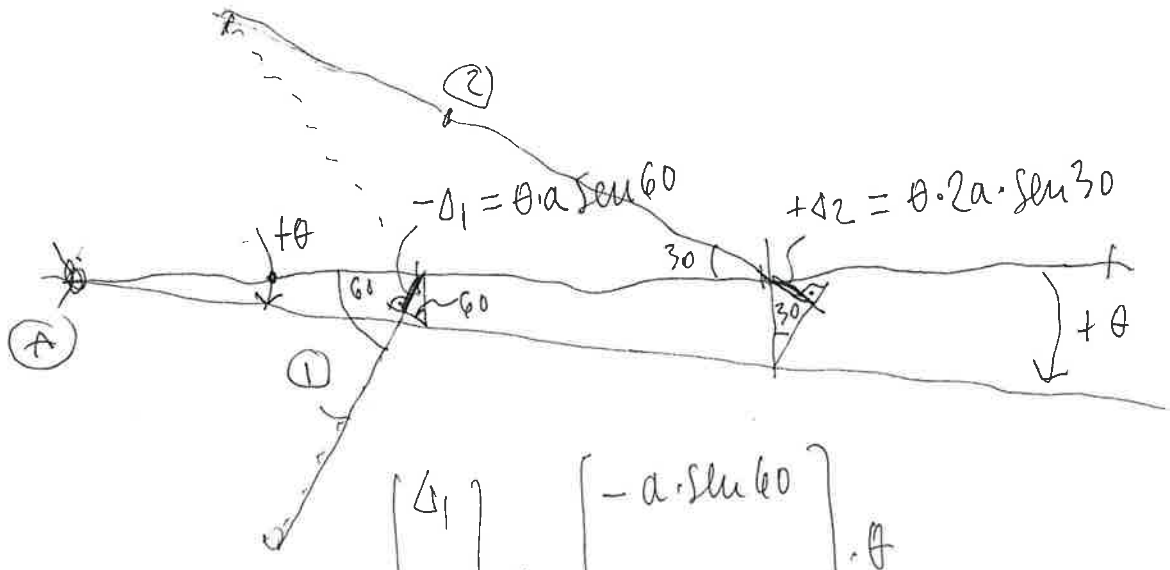


material de ① y ②



$\varepsilon_e = 1{,}3$ ‰ $\Rightarrow E = 200 \frac{kN}{mm^2}$

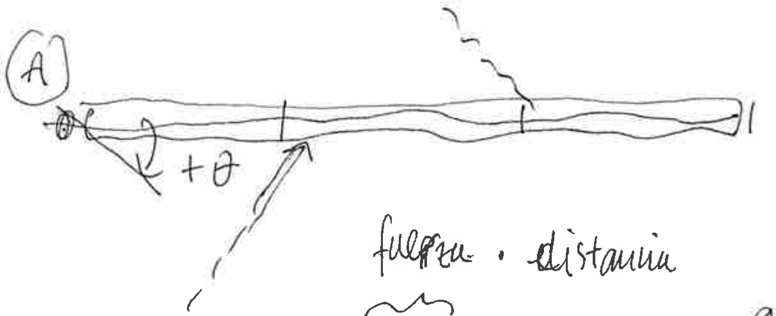
1 solo g.d.l ; p.ej. θ = giro en A



$$\begin{bmatrix} \Delta_1 \\ \Delta_2 \end{bmatrix} = \begin{bmatrix} -a \cdot \text{sen} 60 \\ 2a \cdot \text{sen} 30 \end{bmatrix} \cdot \theta$$

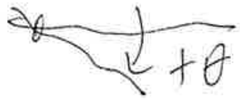
$$\{\Delta\} = [B] \cdot \{\theta\}$$

$\{P\}$ (2) asociado a θ



fuerza · distancia

$$\curvearrowright M = (q\cdot 3\cdot a)\cdot \frac{3a}{2} = \frac{9q\cdot a^2}{2} = \{P\} \qquad \{\delta\} = \theta$$



$$[K] = [B]^T\cdot[K_M]\cdot[B] =$$

$$[K] = \begin{bmatrix} -a\cdot \operatorname{sen}60 , & 2a \operatorname{sen}30 \end{bmatrix} \begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix} \begin{bmatrix} -a\cdot \operatorname{sen}60 \\ 2a \operatorname{sen}30 \end{bmatrix}$$

$$[K] = a^2\cdot \operatorname{sen}^2 60\cdot k_1 + 4a^2\cdot \operatorname{sen}^2 30\cdot k_2$$

ojo unidades, depende de $\{P\}$ y $\{\delta\}$ $\Rightarrow$ $\{P\} = [K]\cdot\{\delta\}$

kN·mm rad

$\Rightarrow$ kN·mm

es conveniente calcular K_1 y K_2 por separado antes de operar en [K], ya que además aparecen en valores más veces.

barra	$E\left(\frac{kN}{mm^2}\right)$	Area (mm^2) $\phi 35$ mm	L (mm)	$K_i = \left(\frac{EA}{L}\right)_i \left(\frac{kN}{mm}\right)$
①	200	962	$5 \cdot 10^3$	$38{,}48 = K_1$
②	200	962	$5\sqrt{3} \cdot 10^3$	$22{,}21 = K_2$

$$[K] = 5000^2 \cdot \operatorname{sen}^2 60 \cdot 38{,}48 + 4 \cdot 5000^2 \cdot \operatorname{sen}^2 30 \cdot 22{,}21 = 1{,}276 \cdot 10^9$$

kN·mm

$$\{P\} = [K] \cdot \{\theta\}$$

$$q \cdot 9 \cdot \frac{5000^2}{2} = 1{,}276 \cdot 10^9 \cdot \frac{\theta}{\text{rad}}$$

en función de q

$$\boxed{\theta = 0{,}088 \cdot q \text{ rad}} \quad \frac{kN}{mm}$$

$$\begin{vmatrix} \Delta_1 \\ \Delta_2 \end{vmatrix} = \begin{vmatrix} -a \operatorname{sen} 60 \\ 2a \operatorname{sen} 30 \end{vmatrix} \theta = \begin{vmatrix} -5000 \cdot \operatorname{sen} 60 \cdot 0{,}088 \cdot q \left(\frac{kN}{mm}\right) \\ +2 \cdot 5000 \cdot \operatorname{sen} 30 \cdot 0{,}088 \cdot q \left(\frac{kN}{mm}\right) \end{vmatrix} \text{mm}$$

~~$\varepsilon_{max} = $~~ $\varepsilon_e = 1{,}3 \cdot 10^{-3}$

¿valor de q para que $\varepsilon_{max} = \varepsilon_e = 1{,}3 \cdot 10^{-3}$?

$$\Delta_{1,max} = 1{,}3 \cdot \cancel{10^{-3}} \cdot 5 \cdot \sqrt{3} \cdot \cancel{10^3} \text{ mm} = 11{,}26 \text{ mm}$$

$$\Delta_{2,max} = 1{,}3 \cdot \cancel{10^{-3}} \cdot 5 \cdot \cancel{10^3} \text{ mm} = 6{,}5 \text{ mm}$$

$$\Rightarrow \begin{vmatrix} q = 2{,}95 \cdot 10^{-2} \frac{kN}{mm} \\ q = 1{,}477 \cdot 10^{-2} \frac{kN}{mm} \end{vmatrix}$$

la más pequeña $\boxed{q = 14{,}77 \frac{kN}{m}}$

en lo que sigue se toma $\underbrace{q = 10 \ \frac{kN}{m}}$

de modo que (10 < 14,77) todos los casos están en régimen elástico.

· de lo antes calculado

$\{P\} = [K]\{\theta\}$

$$q \cdot 9 \cdot \frac{5000^2}{2} = 1{,}276\cdot10^9 \cdot \theta \text{ (rad}$$

$\frac{kN}{mm}$

$$q = 10 \ \frac{kN}{m} = \underbrace{10\cdot10^{-3} \ \frac{kN}{mm}}$$

$$\Rightarrow \theta = 8{,}81\cdot10^{-4} \text{ rad} \equiv 0{,}881 \ \underbrace{\text{miliradianes}}_{\text{mrad}} \equiv 0{,}881 \ \frac{mm}{m}$$

$$\begin{vmatrix} \Delta 1 \\ \Delta 2 \end{vmatrix} = \begin{vmatrix} -5000\cdot \operatorname{sen}60 \\ 2\cdot5000\cdot\operatorname{sen}30 \end{vmatrix} \cdot \theta = \begin{vmatrix} -3{,}815 \\ 4{,}406 \end{vmatrix} mm.$$

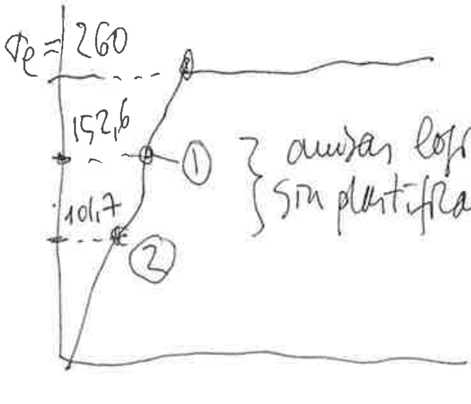
$\rightarrow\square\leftarrow$

$$\begin{vmatrix} N_1 \\ N_2 \end{vmatrix} = \begin{vmatrix} K_1 & 0 \\ 0 & K_2 \end{vmatrix} \begin{vmatrix} \delta_1 \\ \delta_2 \end{vmatrix} = \underbrace{\begin{vmatrix} 38{,}48 & 0 \\ 0 & 22{,}21 \end{vmatrix}}_{\frac{kN}{mm}} \underbrace{\begin{vmatrix} -3{,}815 \\ 4{,}406 \end{vmatrix}}_{mm} = \begin{vmatrix} -146{,}82 \\ 97{,}85 \end{vmatrix} kN$$

$\leftarrow\square\rightarrow$

$$\begin{vmatrix} \sigma_1 \\ \sigma_2 \end{vmatrix} = \begin{vmatrix} \frac{N_1}{A_1} \\ \frac{N_2}{A_2} \end{vmatrix} = \begin{vmatrix} \frac{146{,}82\cdot10^3}{962} \\ \frac{97{,}85\cdot10^3}{962} \end{vmatrix} = \begin{vmatrix} -152{,}6 \\ 101{,}72 \end{vmatrix} \frac{N}{mm^2}$$

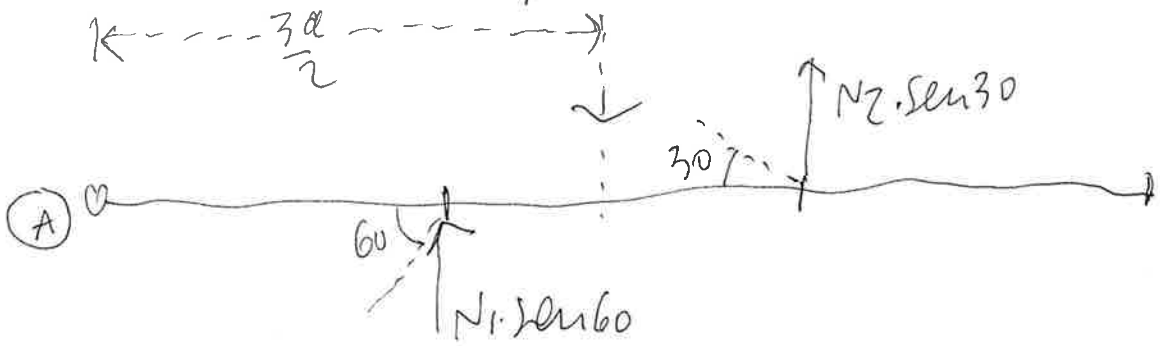
$\left(\frac{N}{mm^2}\right)$



} ambas barras sin plastificar

Comprobamos el equilibrio

$P = q \cdot 3 \cdot a = 10 \cdot 3 \cdot 5 = \underline{150}$ kN



$\Sigma M_{A} = 0$

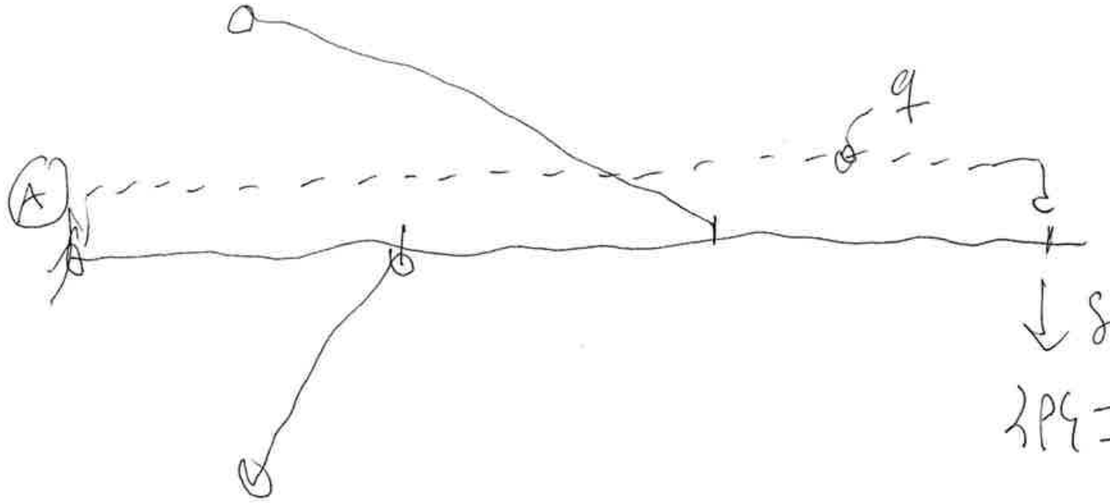
$\Rightarrow \cancel{P} \cdot 150 \cdot \frac{3}{2} \cancel{a} = N_1 \operatorname{sen} 60 \cdot \cancel{a} + N_2 \cdot \operatorname{sen} 60 \cdot 2 \cancel{a}$

$\underbrace{150 \cdot \frac{3}{2}}_{225} = \underbrace{146{,}82 \cdot \operatorname{sen} 60 + 97{,}85 \cdot \operatorname{sen} 30 \cdot 2}_{224{,}9998}$

OK

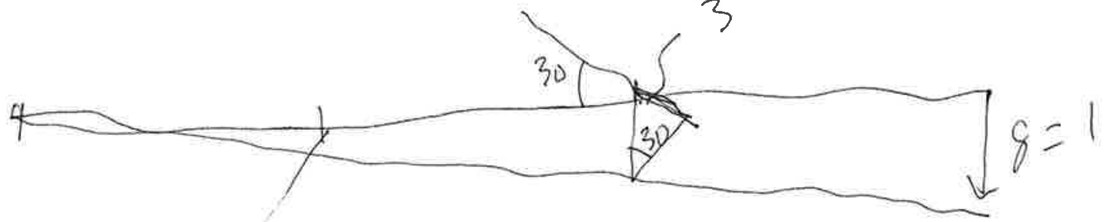
error de arrastre de decimales.

usemos otro g.d.l ; el descenso en punta



$\delta P_4 = \dfrac{q \cdot 3a}{2}$

$$\begin{vmatrix} \delta_1 \\ \delta_2 \end{vmatrix} = \begin{vmatrix} -\frac{1}{3}\,\mathrm{sen}\,60 \\ +\frac{2}{3}\,\mathrm{sen}\,30 \end{vmatrix} \cdot \delta$$



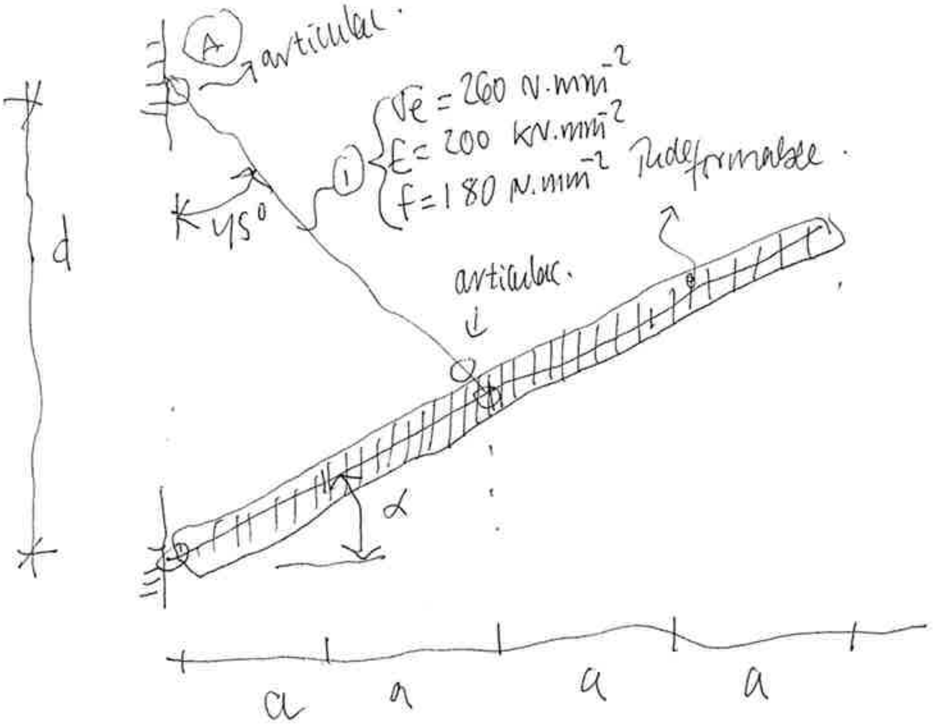
$[B]^T \cdot [K_n] \cdot [B] = 5{,}67 \ \frac{kN}{mm}$

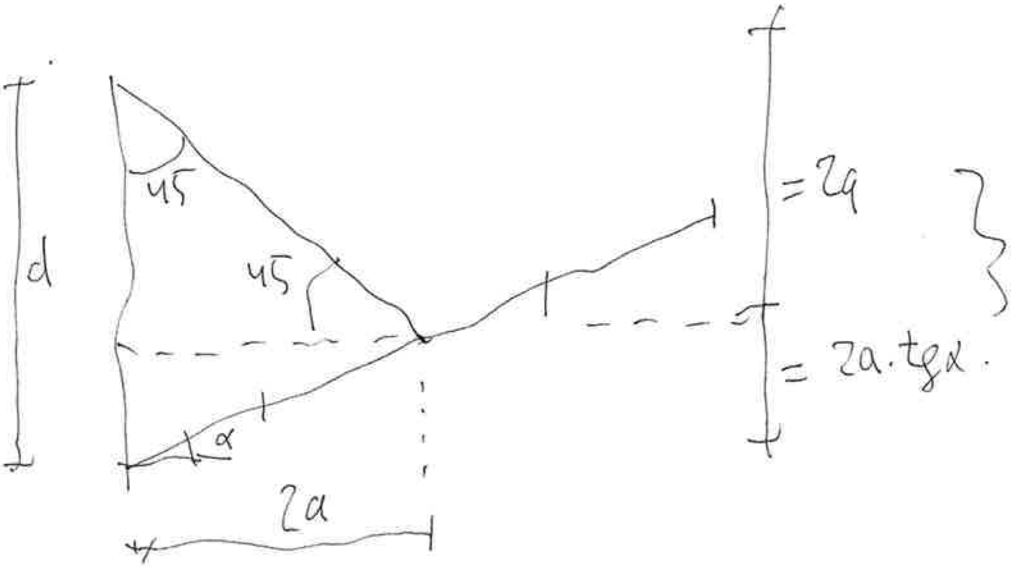
$\delta_4 = [K]^{-1} \cdot \delta P_4 = \frac{1}{5{,}67} \cdot \left(\dfrac{10 \cdot \frac{kN}{m} \cdot 3 \cdot 5\,m}{2} \right) = 13{,}23\ mm$

$$\begin{vmatrix} \delta_1 \\ \delta_2 \end{vmatrix} = [B] \cdot \underbrace{\delta}_{=13{,}23} = \begin{vmatrix} -3{,}819 \\ +4{,}41 \end{vmatrix} mm$$

OK ... pequeñas diferencias por redondeo.

igual que antes

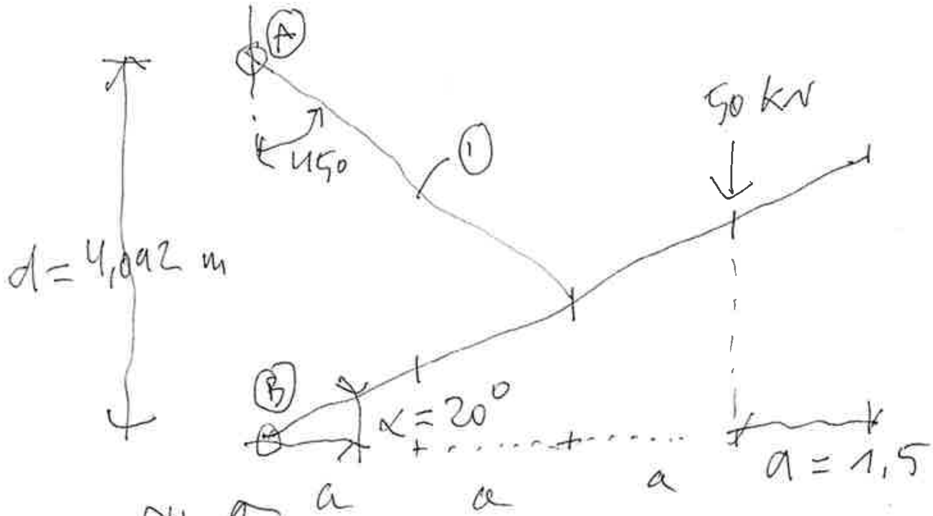


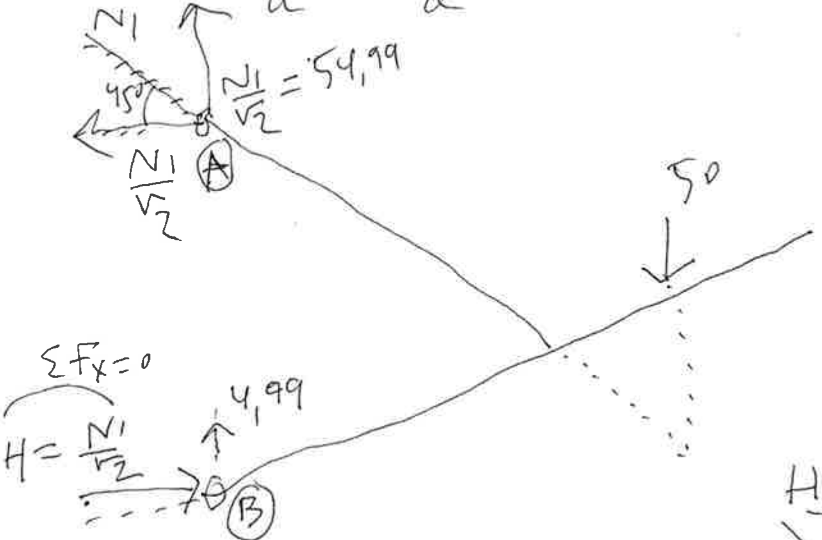


$$2a + 2a \cdot tg\alpha = d$$

$$\frac{d}{(2 + 2 \cdot tg\alpha)} = a$$

Vamos a resolverlo para datos correctos.





$\Sigma M_{B} = 0$ ↺+

$\frac{N_1}{\sqrt{2}} \cdot d = 50 \cdot 3a$

$\underline{H} = \frac{N_1}{\sqrt{2}} = \frac{50 \cdot 3 \cdot 1{,}5}{4{,}092} = \underline{54{,}99\ kN}$

$\underline{N_1} = H \cdot \sqrt{2} = \underline{77{,}76\ kN}$

dimensionado del cable
para $f = 180$

$A = \frac{N}{f} = \frac{77{,}76 \cdot 10^3\ N}{180\ \frac{N}{mm^2}} = 432\ mm^2$

redondeo

si ϕ $A = \frac{\pi \cdot \phi^2}{4} \Rightarrow \phi = \sqrt{\frac{4A}{\pi}} = 23{,}45 \simeq \underline{25\ mm}$ $= \phi$

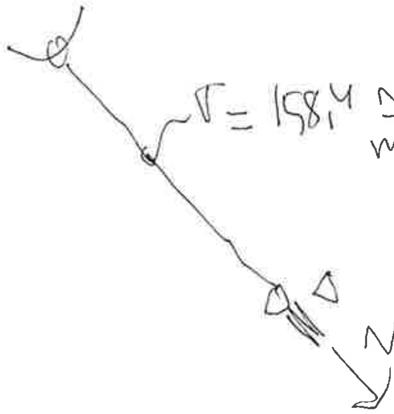
$\underline{\phi = 25\ mm}$

área $\phi 25 \Rightarrow A = \frac{\pi \cdot 25^2}{4} = 490,86$ mm

$A_{min} = 432$ mm

$\sigma_{max} = \underbrace{f}_{=180} \cdot \frac{432}{490.86} = 158,4 \frac{N}{mm}$ ... en el cable.

• ¿cuánto se alarga el cable



$\sigma = 158,4 \frac{N}{mm^2} \rightarrow \underline{\varepsilon} = \frac{\sigma}{E} = \frac{158,4}{200 \cdot 10^3} \frac{N \cdot mm^{-2}}{N \cdot mm^{-2}} = \underline{0,792 \cdot 10^{-3}}$

$N = 77,76$ kN

$\Delta = \varepsilon \cdot L = \varepsilon \cdot \underbrace{2a\sqrt{2}}_{4,24\ m}$

$\Delta = 0,792 \cdot \cancel{10^{-3}}\ 4,24 \cdot \cancel{10^{3}}\ \underline{mm}$

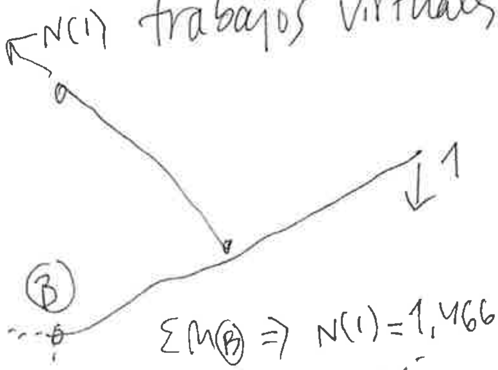
$\underline{\Delta = 3,36\ mm}$

también $N = \left(\frac{EA}{L}\right) \cdot \Delta$

$\hookrightarrow \Delta = \frac{N}{(EA/L)}$

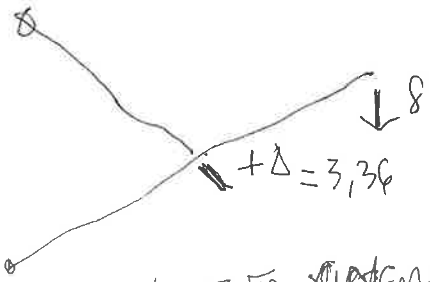
• ¿cuánto desciende el extremo de la barra indeformable?

trabajos virtuales.



$\Sigma M_{(B)} \Rightarrow N(1) = 1,466$

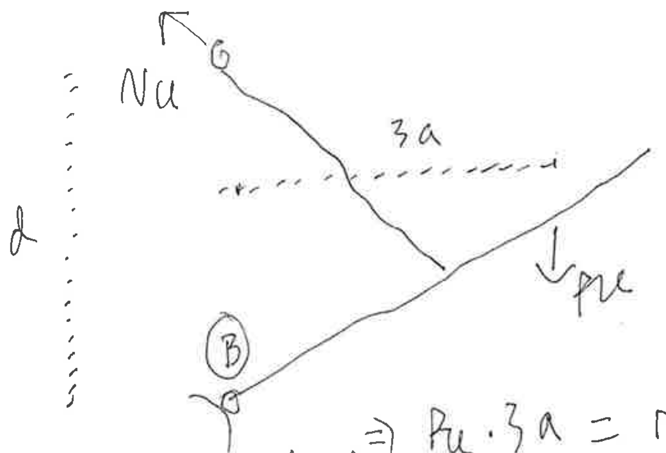
SISTEMA en EQUILIBRIO



MOVIMIENTO ~~VIR~~ CINEMÁTICAMENTE ADMISIBLE

$W_{EXT} = W_{INT}$

$1 \cdot \delta = (+1,466) \cdot (+3,36) = \underline{4,92}$ mm carga última



$\underline{N_u} = \sigma_e \cdot A$

$= 260 \cdot 490{,}86 \cdot 10^{-3}$ kN

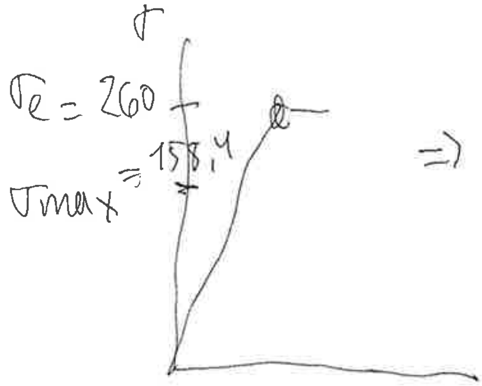
$= 127{,}6$ kN

$\Sigma M_{(B)} = 0 \Rightarrow P_u \cdot 3a = N_u \cdot \cos 45 \cdot d$

$$P_u = \frac{127{,}6 \cdot \frac{1}{\sqrt{2}} \cdot 4{,}092}{3 \cdot 1{,}5} = 82{,}065 \text{ kN}$$

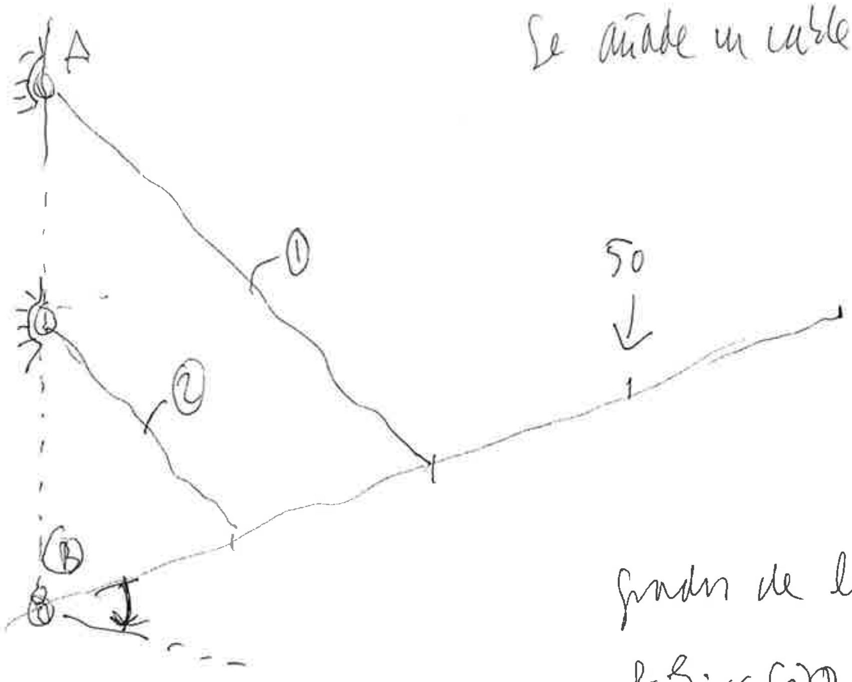
seguridad $\underline{\gamma} = \frac{P_u}{P} = \frac{82{,}06}{50} = \underline{1{,}64}$.

en este caso simplemente se agota el rango elastico ya que es una estr. isostática.



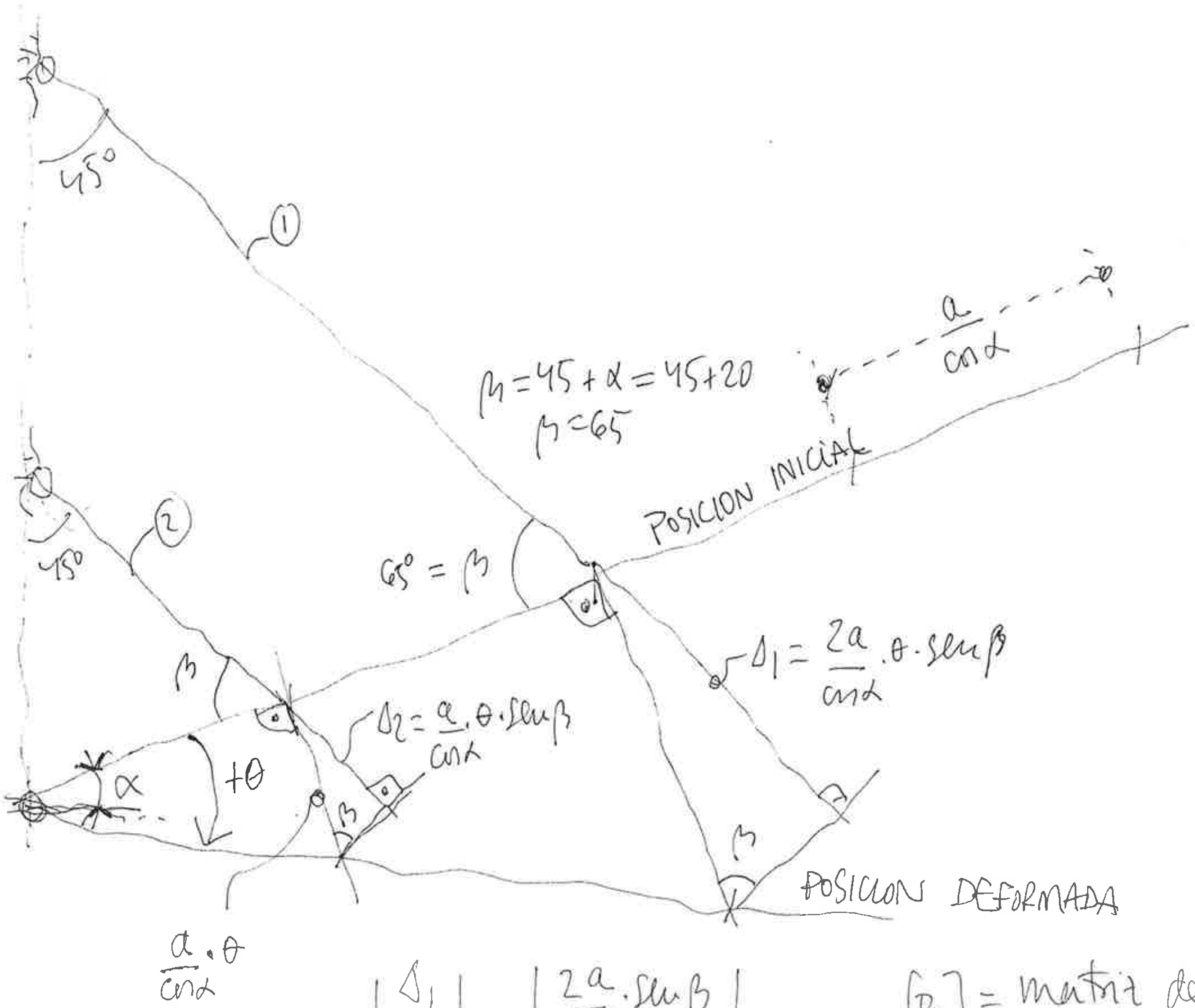
$\Rightarrow P_u = P \cdot \frac{260}{158{,}4} = 50 \cdot \frac{260}{158{,}4} = 82{,}07$ ✓ OK

Se añade un cable, el (2), de igual área y material



grados de libertad (2) = 1

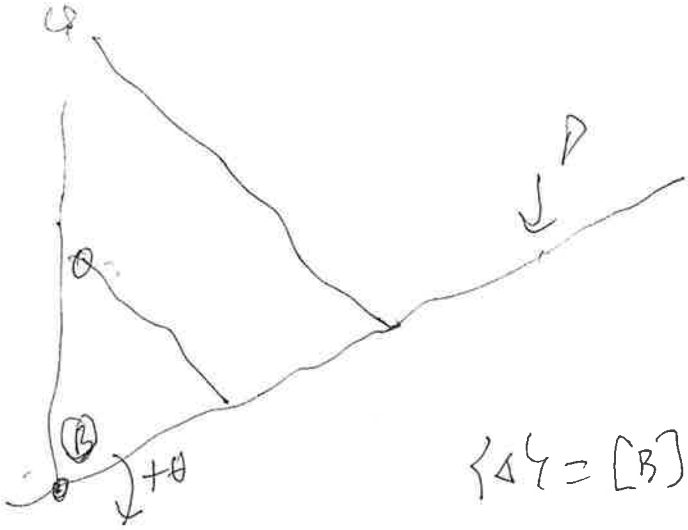
p.ej.: giro en el apoyo B



$$\begin{vmatrix} \Delta_1 \\ \Delta_2 \end{vmatrix} = \begin{vmatrix} \frac{2a}{\cos\alpha} \cdot \operatorname{sen}\beta \\ \frac{a}{\cos\alpha} \cdot \operatorname{sen}\beta \end{vmatrix} \cdot \theta$$

$$\{\Delta\} = [B] \cdot \{\delta\}$$

[B] = matriz de compat.



vector {P} asociado a {δ} = θ

$\{P\} = M_B = \underline{P \cdot 3 \cdot a}$

$\{\Delta\} = [B] \cdot \{\delta\}$

$\{P\} = \underbrace{[B]^T \cdot [K_M] \cdot [B]}_{[K]} \{\delta\}$ $\quad // \ [K_M] = \begin{bmatrix} K_1 & 0 \\ 0 & K_2 \end{bmatrix}$

$[K]$ rigidez de estructura.

$K_i = \left(\frac{EA}{L}\right)_i$ ... rigidez de barra.

$$\underbrace{[\ \cdot \quad \cdot\ ]}_{[B]^T}_{1\times2} \underbrace{\begin{bmatrix} K_1 & \\ & K_2 \end{bmatrix}}_{2\times2} \underbrace{\begin{bmatrix} \cdot \\ \cdot \end{bmatrix}}_{[B]}_{2\times1} = K_1 \cdot \left(\frac{2a}{\cos\alpha} \cdot \operatorname{sen}\beta\right)^2 + K_2 \cdot \left(\frac{a}{\cos\alpha} \cdot \operatorname{sen}\beta\right)^2 = \underset{1\times1}{[K]}$$

$\{P\} = [K] \cdot \{\delta\} \Rightarrow P \cdot 3 \cdot a = [K] \cdot \theta$

$$\theta = \frac{P \cdot 3 \cdot a}{[K]} = \frac{50 \cdot 10^3 \cdot 3 \cdot 1{,}5 \cdot 10^3}{\underbrace{\frac{200 \cdot 10^3}{4{,}24 \cdot 10^3} \cdot \frac{\pi \cdot 25^2}{4}}_{K_1} \cdot \left(\frac{2 \cdot 1{,}5 \cdot 10^3}{\cos 20} \cdot \operatorname{sen} 65\right)^2 + \underbrace{\frac{200 \cdot 10^3}{2{,}12 \cdot 10^3} \cdot \frac{\pi \cdot 25^2}{4}}_{K_2} \cdot \left(\frac{1{,}5 \cdot 10^3}{\cos 20} \cdot \operatorname{sen} 65\right)^2}$$

$\underline{\theta = 0{,}774 \cdot 10^{-3} \text{ rad}} \equiv 0{,}774$ miliradianes $= $ m.rad $\equiv 0{,}774 \ \frac{mm}{m}$

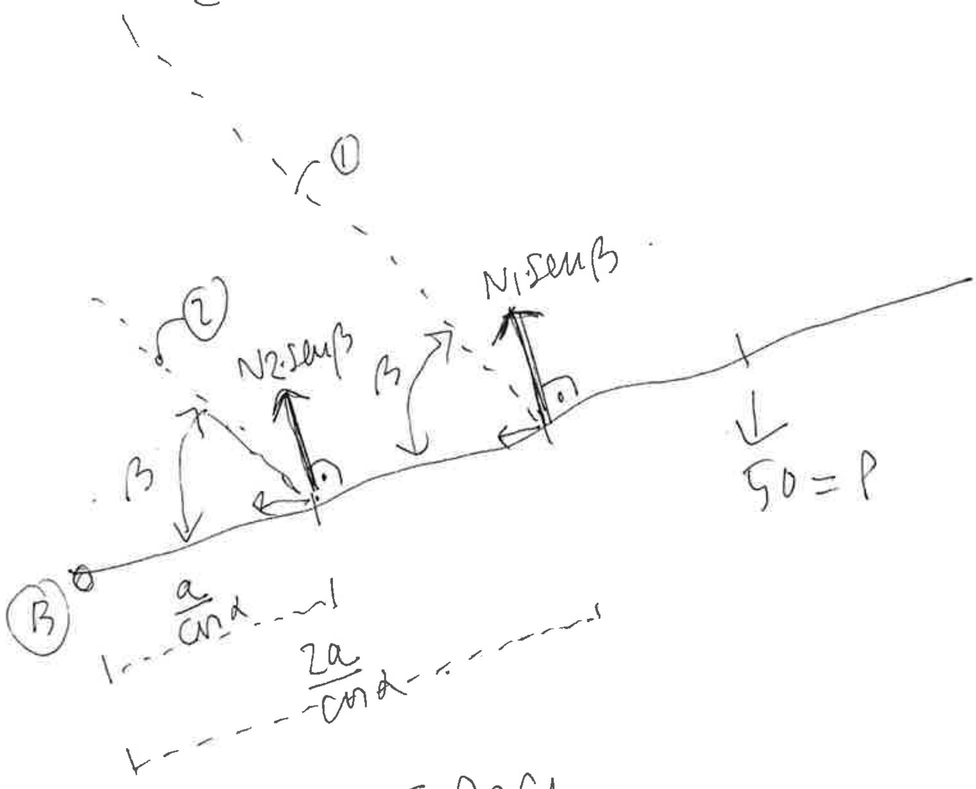
↻ + horario

$[B]^T$ es la <u>matriz de equilibrio</u>.

$\{P\} = [B]^T \cdot \{N\}$. $\qquad \{P\} = P \cdot 3 \cdot a$

$[B]^T = \left[\frac{2a}{\cos\alpha} \cdot \text{sen}\beta \; , \; \frac{a}{\cos\alpha} \cdot \text{sen}\beta \right] \qquad \{N\} = \begin{bmatrix} N_1 \\ N_2 \end{bmatrix}$

¿como se entiende esta relación?



$\Sigma M_{(B)} = 0$

$$\Rightarrow N_1 \cdot \text{sen}\beta \cdot \frac{2a}{\cos\alpha} + N_2 \cdot \text{sen}\beta \cdot \frac{a}{\cos\alpha} = \overbrace{P \cdot 3 \cdot a}^{\{P\}}.$$

$$\Rightarrow \underbrace{\{P\}}_{} = P \cdot 3a = \underbrace{\left[\frac{2a}{\cos\alpha} \cdot \text{sen}\beta \; , \; \frac{a}{\cos\alpha} \cdot \text{sen}\beta \right]}_{[B]^T} \cdot \underbrace{\begin{bmatrix} N_1 \\ N_2 \end{bmatrix}}_{\{N\}}$$

alargamientos.

$$\{D\} = [B]\cdot\{\delta\} \Rightarrow \begin{bmatrix} \Delta_1 \\ \delta_2 \end{bmatrix} = \begin{bmatrix} \frac{2a}{\cos\alpha}\cdot \operatorname{sen}\beta \\ \frac{a}{\cos\alpha}\cdot \operatorname{sen}\beta \end{bmatrix}\cdot\theta$$

$$\underbrace{\begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix}}_{} = \begin{bmatrix} \frac{2\cdot 1{,}5\cdot 10^3\cdot \operatorname{sen}65}{\cos\cdot 20} \\ \frac{1{,}5\cdot 10^3\cdot \operatorname{sen}65}{\cos 20} \end{bmatrix}\cdot \underbrace{0{,}773\cdot 10^{-3}}_{rad} = \underbrace{\begin{bmatrix} 2{,}24 \\ 1{,}12 \end{bmatrix}}_{} mm$$

Axiles o normales.

$$\begin{bmatrix} N_1 \\ N_2 \end{bmatrix} = \begin{bmatrix} K_1 & \\ & K_2 \end{bmatrix}\cdot\begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix} = \overbrace{\begin{bmatrix} 23153{,}7 & 0 \\ 0 & 46307{,}5 \end{bmatrix}}^{N/mm} \overbrace{\begin{bmatrix} 2{,}24 \\ 1{,}12 \end{bmatrix}}^{mm} = \overbrace{\begin{bmatrix} 51{,}86\cdot 10^3 \\ 51{,}86\cdot 10^3 \end{bmatrix}}^{Newtons}$$

$$\{N\} = \widetilde{[KM]}\cdot\{D\}$$

$$\Rightarrow N_1 = N_2 = \underline{51{,}86}\ \underline{kN}$$

¿por qué tienen el mismo axil?

$$\varepsilon_i = \frac{\Delta_i}{L_i} \Rightarrow \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix} = \begin{bmatrix} \Delta_1/L_1 \\ \Delta_2/L_2 \end{bmatrix} = \begin{bmatrix} \frac{2{,}24}{4{,}24\cdot 10^3} \\ \frac{1{,}12}{2{,}12\cdot 10^3} \end{bmatrix} = \begin{bmatrix} 0{,}528\cdot 10^{-3} \\ 0{,}528\cdot 10^{-3} \end{bmatrix} \Rightarrow \underbrace{\varepsilon_1 = \varepsilon_2 = 0{,}528\cdot 10^{-3}}$$

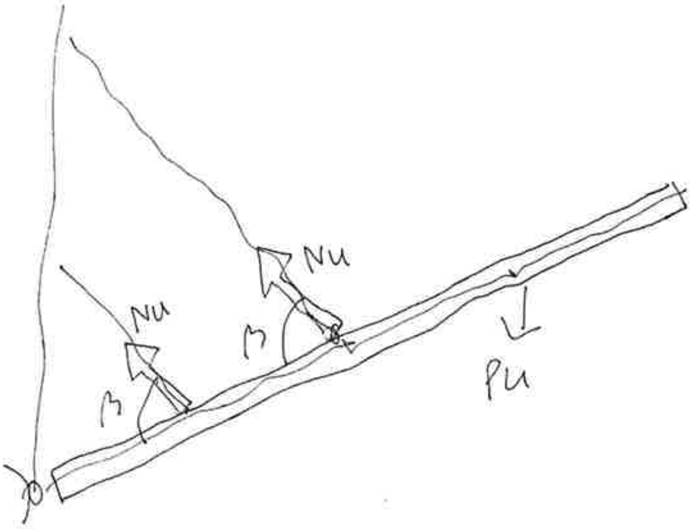
y además

$$\underbrace{A_1 = A_2}$$

iguales áreas.

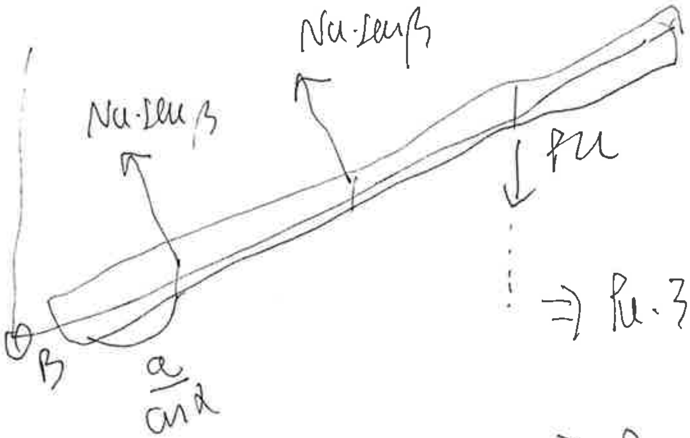
$$\Rightarrow \underline{N_1 = N_2}$$

Carga ultima.



$\begin{cases} \sigma_e = 260 \ N \cdot mm^{-2} \\ \phi 25 \end{cases}$

$N_u = \sigma_e \cdot A = 127{,}6$ kN



$\sum M_{(B)} = 0$

$\Rightarrow P_u \cdot 3 \cdot a = N_u \operatorname{sen}\beta \left(\frac{a}{\cos\alpha} + \frac{2a}{\cos\alpha} \right)$

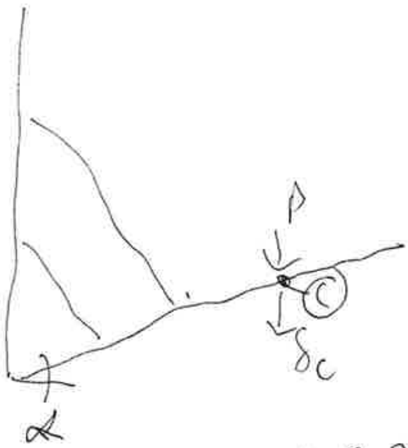
$\Rightarrow P_u = 123{,}1$ kN

Seguridad $\gamma = \frac{P_u}{P} = \frac{123{,}1}{50} = 2{,}46$

mayor que en el caso Isostático.

consideremos otros g.d.l.

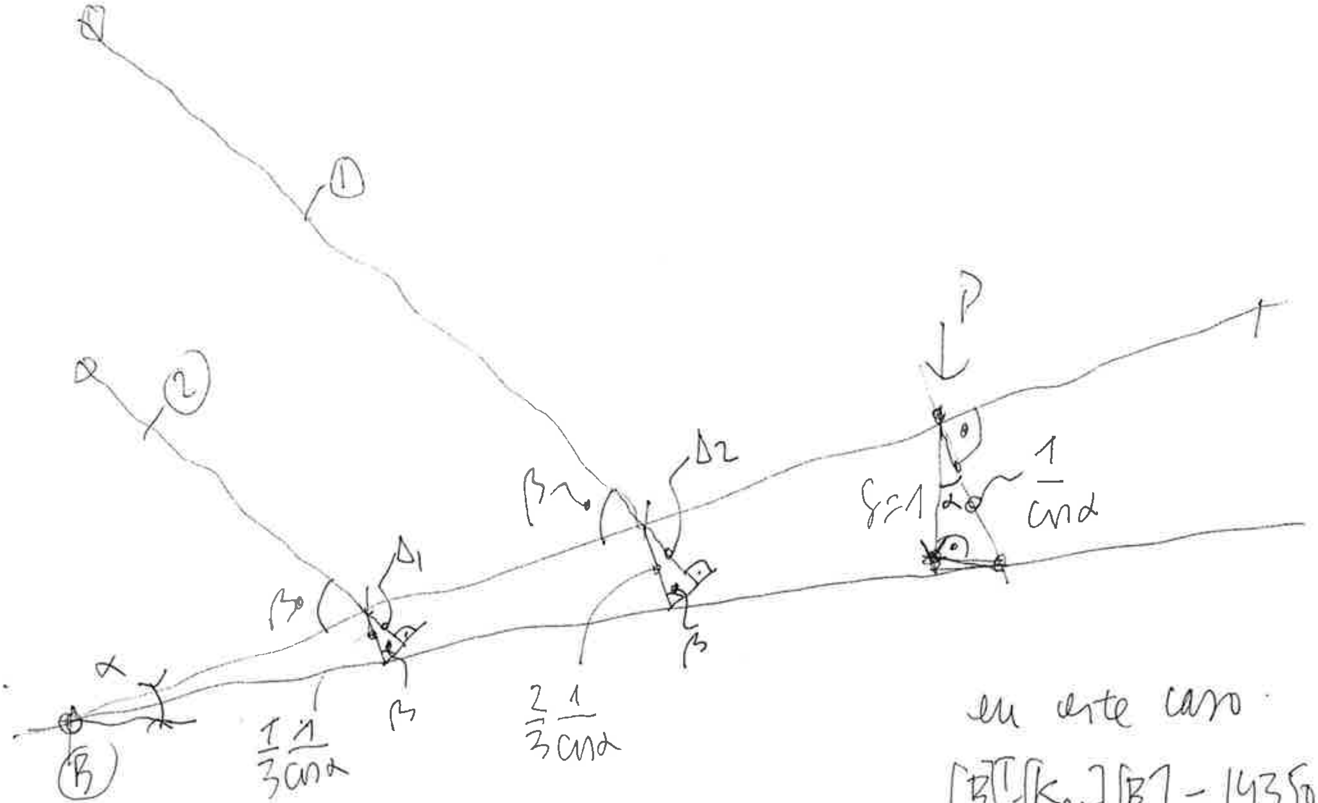
el más sencillo es tomar como $\{\delta\}$
el descenso en el punto de aplicación de P



$\{\delta\} = \delta_C$

$\{P\} = P$

¿cómo es $[B]$?



en este caso

$[B]^T [K_m] [B] = 14350 \ \frac{N}{mm}$

$\delta_C = \frac{P}{K} = \frac{50 \cdot 10^3}{14350} = \underline{3{,}48 \ mm}$

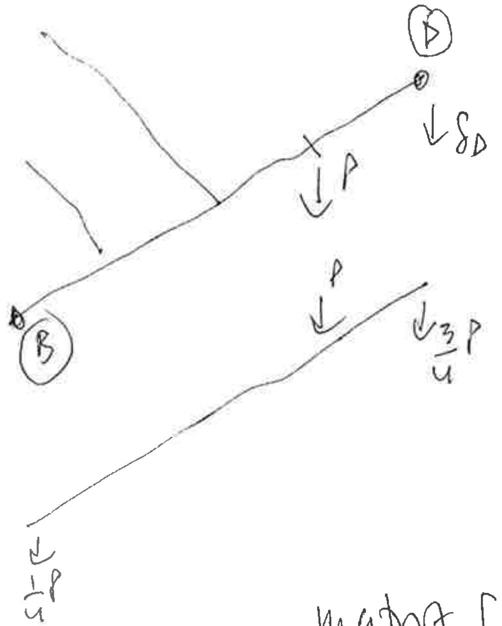
$$\begin{Bmatrix} \Delta_1 \\ \Delta_2 \end{Bmatrix} = \begin{Bmatrix} \frac{2}{3}\frac{1}{\cos\alpha}\cdot \operatorname{sen}\beta \\ \frac{1}{3}\frac{\operatorname{sen}\beta}{\cos\alpha} \end{Bmatrix} \cdot \underbrace{\delta_C}_{3{,}48}$$

$$\{\Delta\} = \underbrace{[B]} \cdot \{\delta\}$$

$$\Rightarrow \begin{vmatrix} \Delta_1 = 2{,}24 \\ \Delta_2 = 1{,}12 \end{vmatrix} \ mm$$

OK

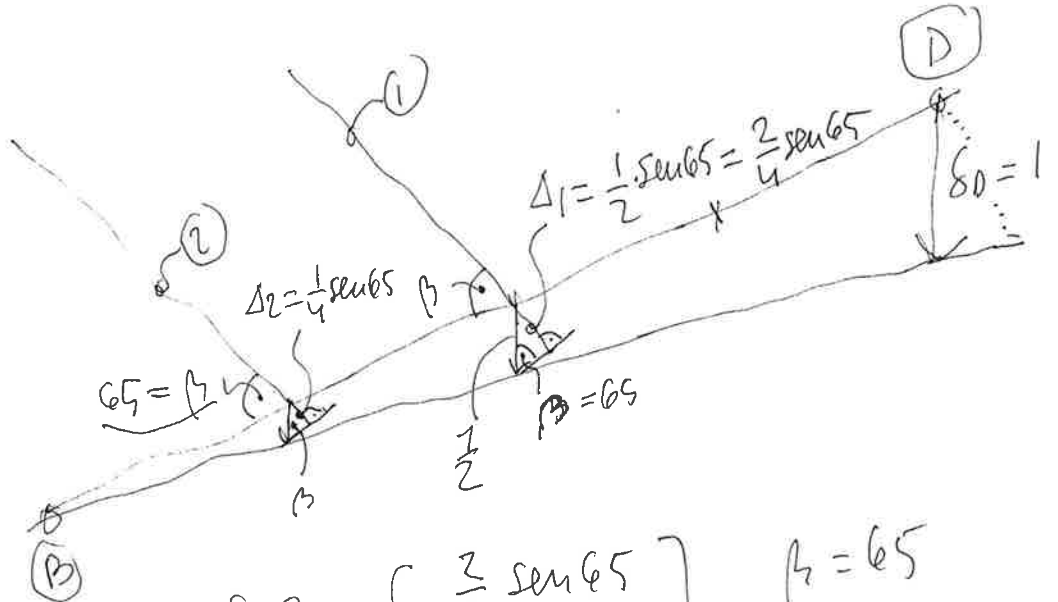
Si se usa el descenso en el ~~extr~~ extremo ya no es tan obvio



$\Rightarrow$ $\{\delta\} = \delta_D$

$\{P\} = \frac{3}{4}P$

matriz [B].



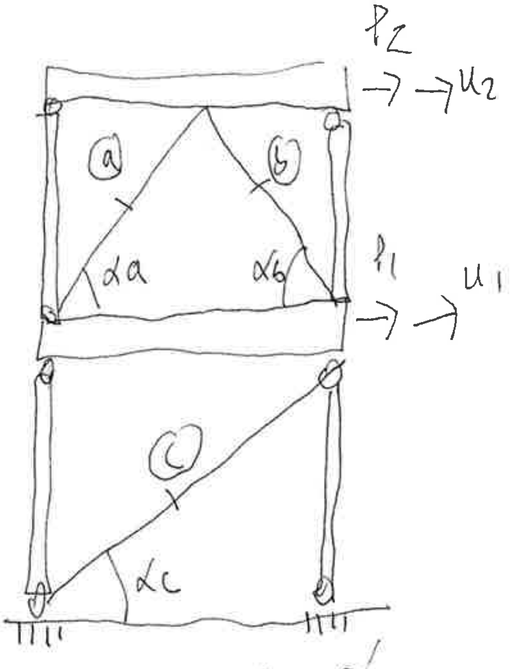
$$[B] = \begin{pmatrix} \frac{2}{3}\,\text{sen}65 \\ \\ \frac{1}{4}\,\text{sen}65 \end{pmatrix} \qquad \beta = 65$$

$\{P\} = [K]\cdot\{\delta\} \Rightarrow P = (K)\cdot \delta_D$

$[K] = [B]^T\cdot[K_{n}][B] = \cancel{10380}\ \frac{N}{mm}$ $8071,88$

$\delta_D = \frac{P}{(K)} = \frac{50\cdot 10^3}{\cancel{10380}\ 8071,88} = \cancel{4,81}\ \underline{4,645}\ mm$

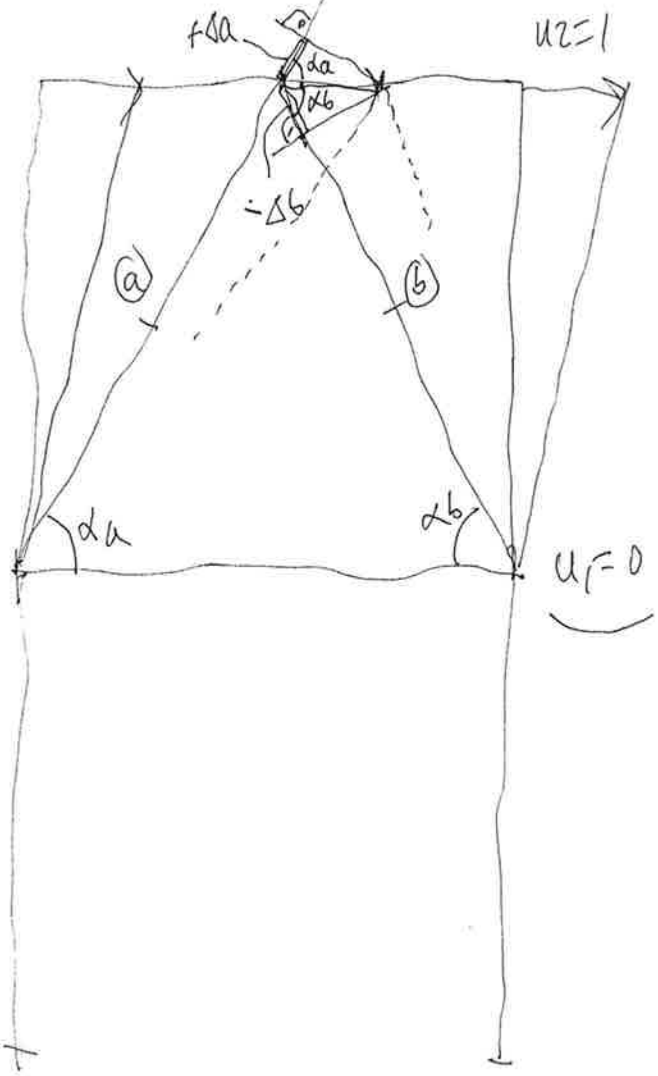
$\Rightarrow \{\Delta\} = [B]\cdot\delta_D \Rightarrow \begin{vmatrix} \Delta 1 = 2,24 \\ \Delta 2 = 1,12 \end{vmatrix} mm$ OK., igual



ⓐ, ⓑ y ⓒ barras deformables, biarticuladas
el resto indeformable.

$$\{P\} = \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} \qquad \{\delta\} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

2 g.d.l



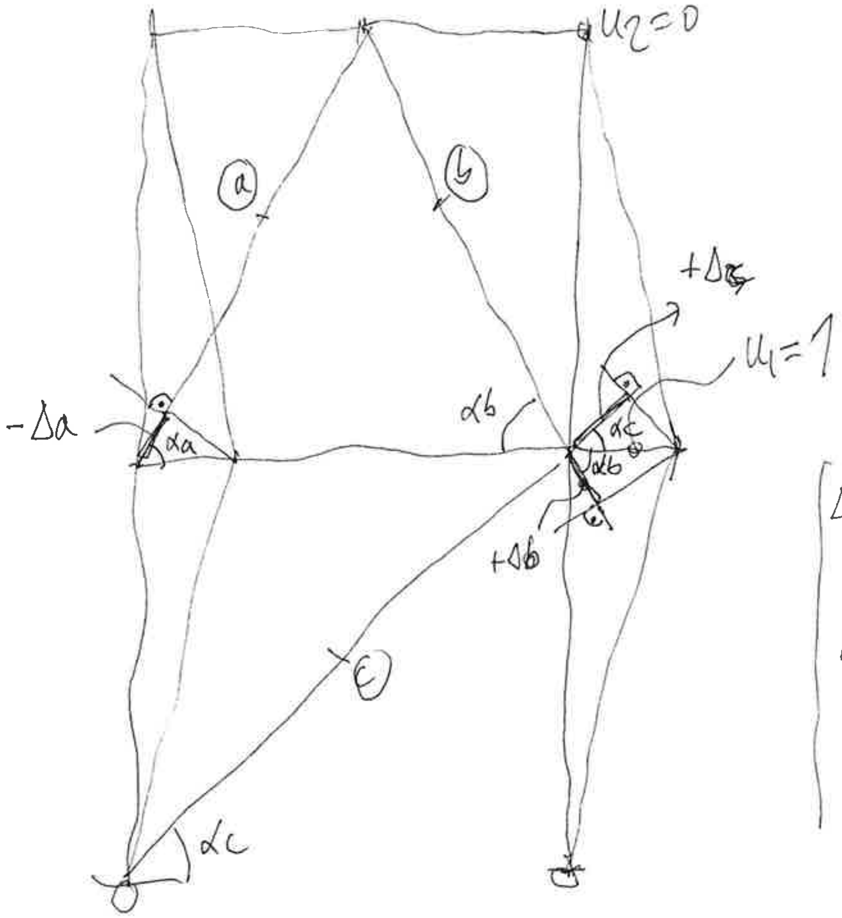
$$+\Delta a = u_2 \cdot \cos \alpha_a$$

$$-\Delta b = u_1 \cdot \cos \alpha_b$$

segunda columna de [B]

$$\Rightarrow \begin{vmatrix} \Delta a \\ \Delta b \\ \delta c \end{vmatrix} = \begin{vmatrix} \cos \alpha_a \\ -\cos \alpha_b \\ 0 \end{vmatrix} \begin{vmatrix} u_1 \\ u_2 \end{vmatrix}$$

$$\{\Delta\} = \underbrace{[B]} \cdot \{\delta\}$$



$$-\Delta_a = u_1 \cdot \cos\alpha_a$$
$$+\Delta_b = u_1 \cos\alpha_b$$
$$+\Delta_c = u_1 \cos\alpha_c$$

primera columna

$$\underbrace{\begin{vmatrix} \Delta_1 \\ \delta_2 \\ \Delta_3 \end{vmatrix} = \begin{vmatrix} -\cos\alpha_a & \\ \cos\alpha_b & \\ \cos\alpha_c & \end{vmatrix}}_{[B]} \begin{vmatrix} u_1 \\ u_2 \end{vmatrix}$$

es decir, considerando ambos casos juntos

$$[B] = \begin{vmatrix} -\cos\alpha_a & \cos\alpha_a \\ \cos\alpha_b & -\cos\alpha_b \\ \cos\alpha_c & 0 \end{vmatrix}$$

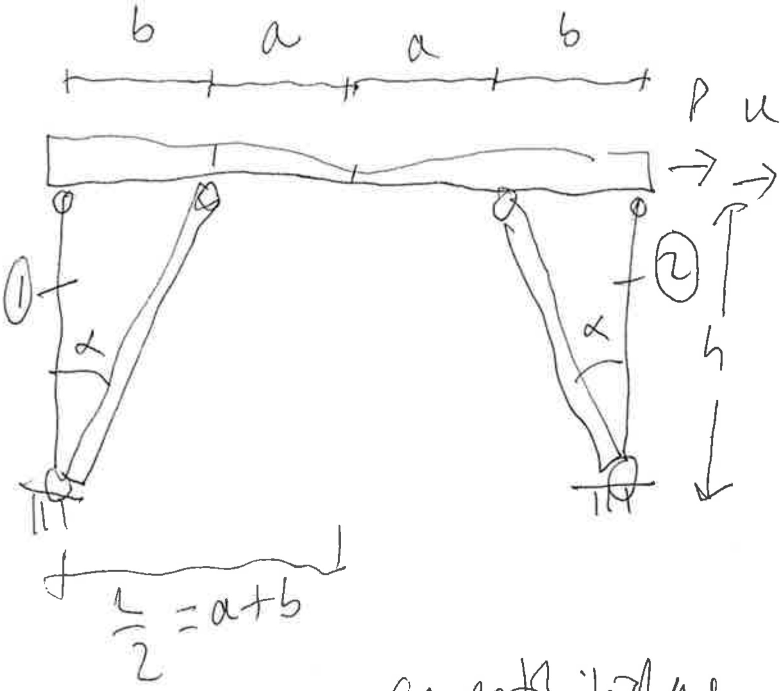
siendo además

$$[K_M] = \begin{bmatrix} K_a & \cdots & 0 \\ \cdots & K_b & \cdots \\ 0 & \cdots & K_c \end{bmatrix}$$

$K_i = \left(\frac{EA}{L}\right)_i$: rigidez de barra

$$[K] = [B]^T \cdot [K_M] \cdot [B]$$

$$\left.\{P\} = \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} \;;\; \{\delta\} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}\right\} \quad \{P\} = \underbrace{[B]^T \cdot [K_M] \cdot [B]}_{[K] = \text{rigidez de estructura}} \cdot \{\delta\}$$

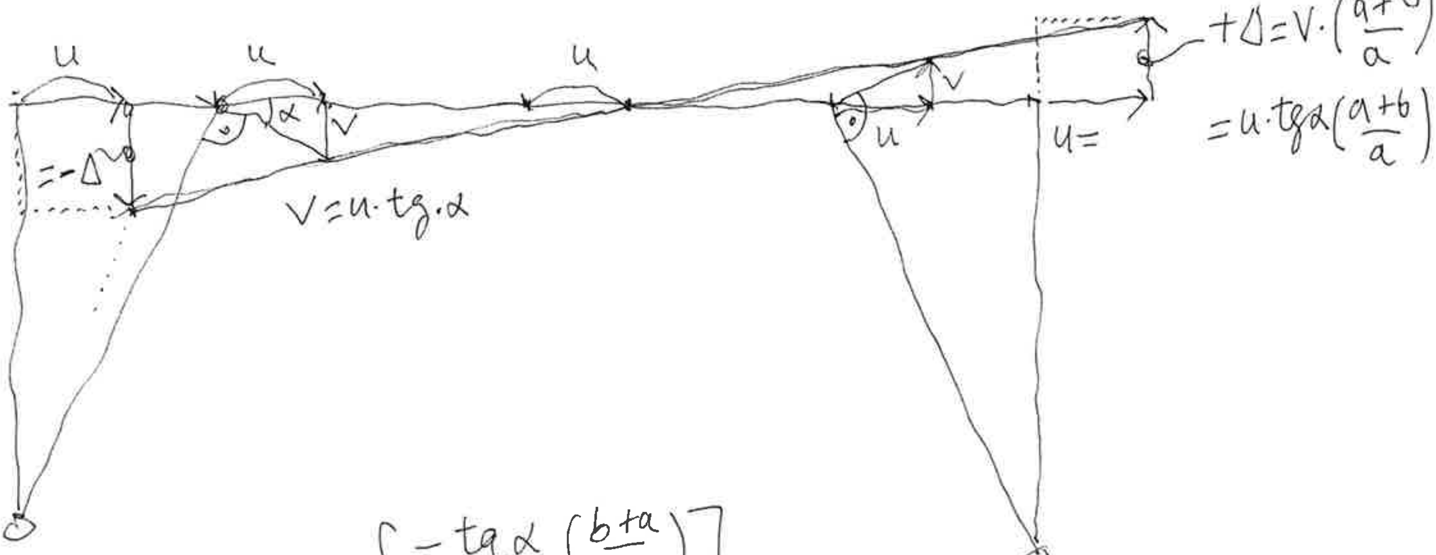


① y ② deformables
resto indeformable.

3ES = P

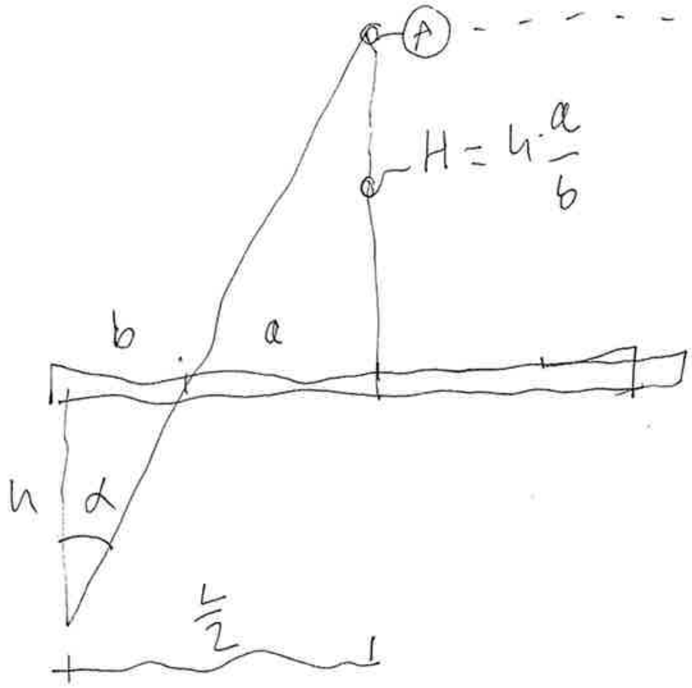
3ES = u ... 1 g.d.l.

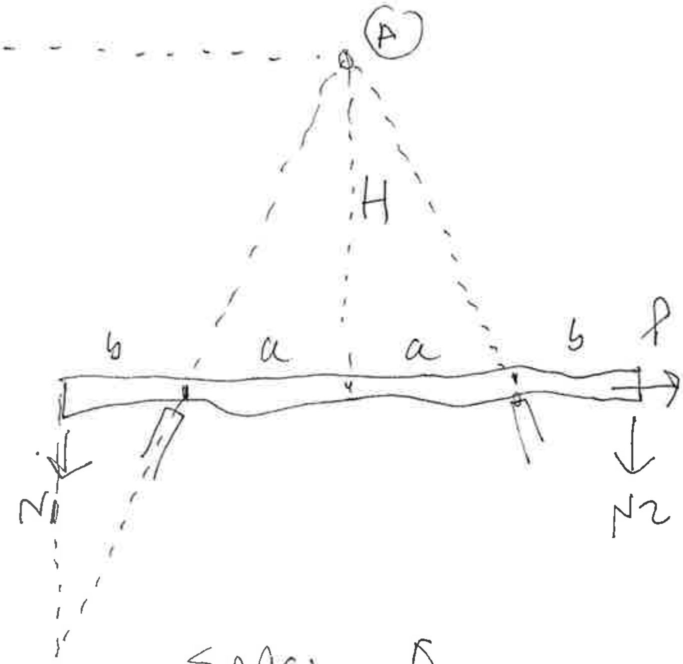
Compatibilidad



$$[B] = \begin{bmatrix} -tg\alpha\left(\frac{b+a}{a}\right) \\ +tg\alpha\left(\frac{b+a}{a}\right) \end{bmatrix} \cdot u$$

¿Que situación de equilibrio conduce a una $[B]^T$ asociada a la anterior.





$$\Sigma M_{(A)} = 0$$

$$P \cdot H + N_1 \cdot \frac{L}{2} - N_2 \cdot \frac{L}{2} = 0$$

$$P = -N_1 \cdot \frac{L}{2H} + N_2 \cdot \frac{L}{2H}$$

$$\{P\} = P \quad \{N\} = \begin{bmatrix} N_1 \\ N_2 \end{bmatrix} \Rightarrow \quad P = \underbrace{\left[-\frac{L}{2H}, \frac{L}{2H}\right]} \begin{bmatrix} N_1 \\ N_2 \end{bmatrix}$$

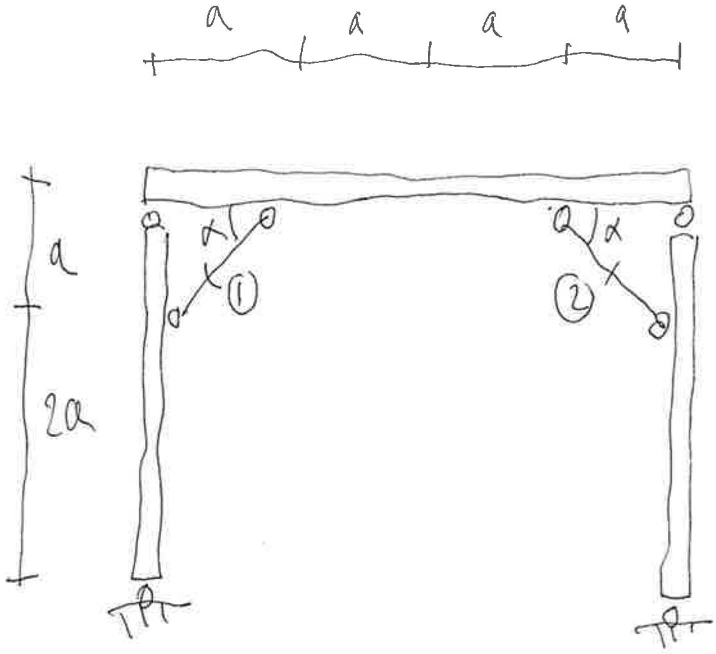
$$\{P\} = [B]^T \cdot \{N\}$$

pero reorganizando $[B]^T$.

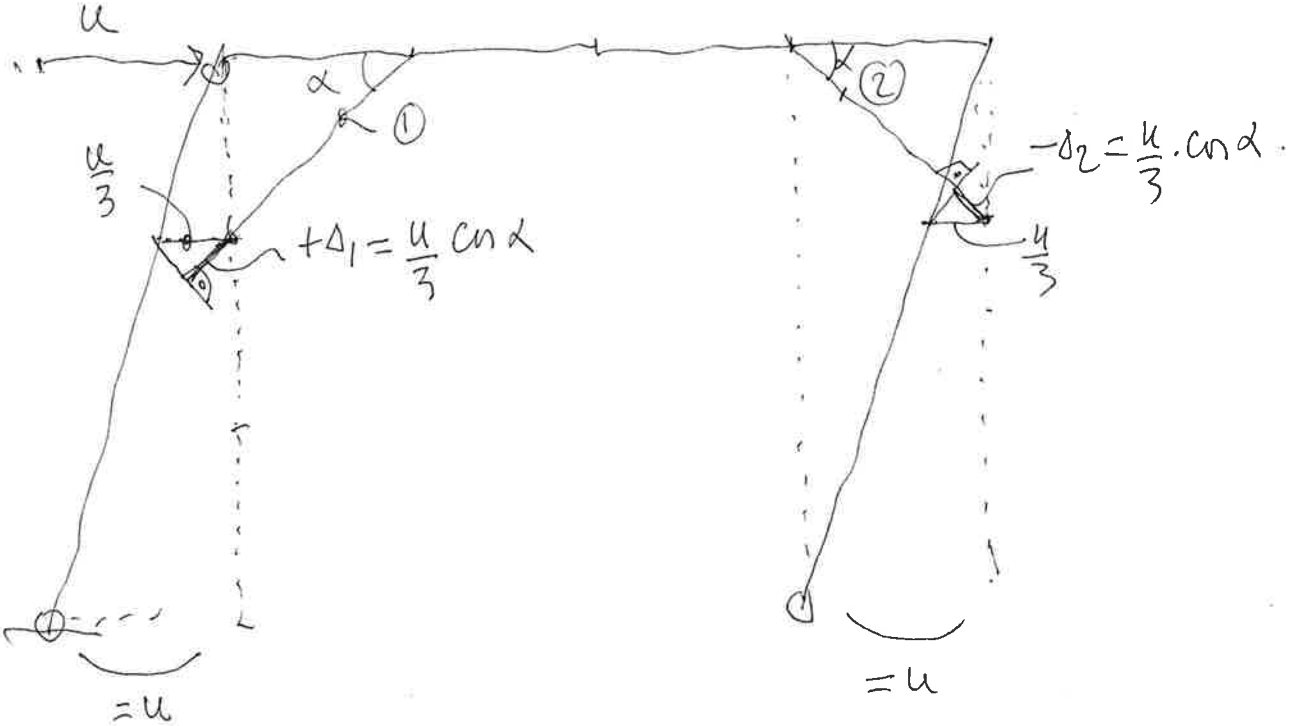
$$\left.\begin{array}{l} \frac{L}{2} = a + b \\ H = h \cdot \frac{a}{b} \end{array}\right\} \Rightarrow \frac{L}{2} \frac{1}{H} = (a+b) \cdot \underbrace{\frac{b}{a \cdot h}}_{tg\alpha} = tg\alpha \frac{(a+b)}{a}$$

$$\Rightarrow [B]^T = \left[-tg\alpha \cdot \frac{(a+b)}{a}, + tg\alpha \frac{(a+b)}{a}\right]$$

como antes.



1 y 2 deformable,
el resto indeformable.

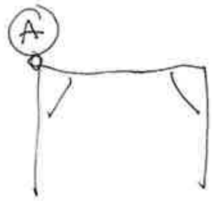


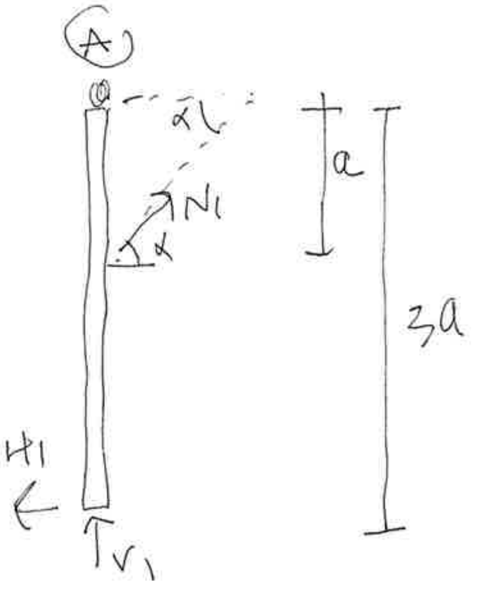
$$\begin{vmatrix} \Delta_1 \\ \delta_2 \end{vmatrix} = \underbrace{\begin{vmatrix} \frac{1}{3}\cos\alpha \\ -\frac{1}{3}\cos\alpha \end{vmatrix}} \cdot u$$

$$\{\Delta\} = [B] \cdot \{\delta\}$$

obteniendo $\begin{vmatrix} N_1 \\ N_2 \end{vmatrix}$ ...

por equilibrio de soportes ...



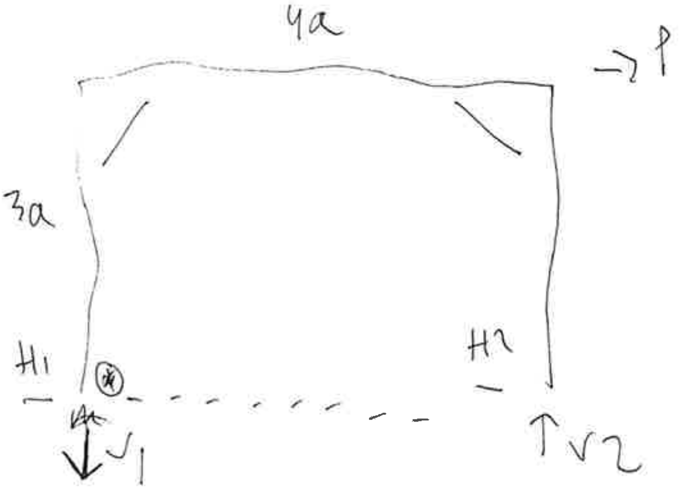


$$\Sigma M_{A} = 0$$

$$N_1 \cdot \cos\alpha \cdot a = H_1 \cdot 3a$$

$$H_1 = \frac{N_1 \cdot \cos\alpha}{3}$$

... de forma similar H_2 ...



V_1 y V_2 salen por equilibrio global directamente (sin depender de los empujes)

$$\Rightarrow \left.\begin{array}{l} P \cdot 3a = V_2 \cdot 4a \\ P \cdot 3a = V_1 \cdot 4a \end{array}\right\} V_1 = V_2 = \frac{P\, 3\cancel{a}}{4\cancel{a}} = \frac{3}{4} P$$

... con el sentido indicado